

Ground-State Properties for Coupled Bose-Einstein Condensates inside a Cavity Quantum Electrodynamics

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We analytically investigate the ground-state properties of two-component Bose-Einstein condensates with few ^{87}Rb atoms inside a high-quality cavity quantum electrodynamics. In the $\text{SU}(2)$ representation for atom, this quantum system can be realized a generalized Dicke model with a quadratic term arising from the interatomic interactions, which can be controlled experimentally by Feshbach resonance technique. Moreover, this weak interspecies interaction can give rise to an important zero-temperature quantum phase transition from the normal to the superradiant phases, where the atomic ensemble in the normal phase is collectively unexcited while is macroscopically excited with coherent radiations in the superradiant phase. Finally, we propose to observe this predicted quantum phase transition by measuring the direct and striking signatures of the photon field in terms of a heterodyne detector out of the cavity.

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Cavity quantum electrodynamics has an essential microwave photon field to explore the important microscopic quantum phenomena in quantum optics since the strong atom-field coupling strength, which dominates over the dissipative losses of the quantum system, has been achieved[1, 2, 3]. Recently, much progress has been made in the observation of the motional dynamics[4, 5] as well as the trapping[6, 7] and cooling[8, 9] of single atom within the cavity mode. This also provides an interesting way to process quantum information and implement quantum computing such as nonclassical light sources[10, 11] and quantum state transfer[12].

Recently, Bose-Einstein condensate (BEC) coupled with a high-quality cavity quantum electrodynamics has been attracted much attentions in both theory and experiment. Despite BEC is a central goal for atom chips, it exhibits some novel macroscopic quantum effects, which has never predicted and observed in traditional atom. For example, atom-atom or atom-photon entanglement and squeezing can be produced in the linear regime of collective atomic recoil lasing, where the ground state of the condensate remains approximately undepleted[13, 14, 15]. By solving the three-mode master equation in the Wigner representation, three-mode entanglement as well as two-mode atom-atom and atom-radiation entanglement can be generally robust against losses and decoherence, and thus regarded as a good candidate for the experimental observation of entanglement in condensate systems[16]. Moreover, a novel cavity-mediated long-range atom-atom interactions has been also realized theoretically[17]. However, the experimental process has only been considered recently [18, 19, 20, 21, 22, 23]. The main difficulties arise from adverse vacuum requisites and sophisticated topological requirements on both of these art technologies. Here we theoretically discuss two-component BECs interacting with a high-quality cavity quantum electrodynam-

ics. It has been shown that in two-component BECs, the weak nonlinear interactions controlled experimentally by Feshbach resonance technique[24] can give rise to a number of novel phenomena including instability[25], metastability[26], wave mixing[27], soliton[28, 29, 30], dynamical bifurcation[31], chaos[32, 33], and Heisenberg-limited Mach-Zahnder interferometry[34]. Very recently, a novel second-order quantum phase transition from the normal to the tunneling phases has been predicted in the resonant case when two-component BECs are coupled with a periodically driven laser field[35].

In this paper we mainly investigate the ground-state properties of two-component BECs with few ^{87}Rb atoms inside an ultrahigh finesse optical cavity, which can support a single-mode photon. We show that in the $\text{SU}(2)$ representation for atom, this quantum system can be realized a generalized Dicke model with a quadratic term arising from the mean-field interatomic interactions. By means of Holstein-Primakoff transformation [36] and boson expansion method[37], the ground-state energy can be approximately evaluated and show that the weak interspecies interaction can lead to an important zero-temperature quantum phase transition from the normal to the superradiant phases, where the atomic ensemble in the normal phase is collectively unexcited while is macroscopically excited with coherent radiations in the superradiant phase. Finally, we propose to observe this predicted quantum phase transition by detecting the direct and striking signatures of the photon field such as the well-measured intracavity intensity $I \propto |\langle a \rangle|^2$ in terms of a heterodyne detector out of the cavity[38]. Moreover, its essential features of quantum criticality such as the scaling behavior, critical exponent and universality are also given.

A physical realization shown in Fig.1 is that two condensates in different hyperfine levels $|F=1, m_f=-1\rangle$ ($|1\rangle$) and $|F=2, m_f=1\rangle$ ($|2\rangle$) are confined in a time-

average, orbiting potential magnetic trap. Usually, in the ^{87}Rb system, an external laser is applied to induce a Josephson-like coupling and the detuning of the laser is adiabatically changed to produce various transitions between $|1\rangle$ and $|2\rangle$. However, here this tunable laser can be displaced by the quantized field of the cavity mode after two condensates are sent into the optical cavity. For a high-quality cavity with the length $178\text{ }\mu\text{m}$ and the mode waist radius $26\text{ }\mu\text{m}$, the finesse is given by 3×10^5 and the maximum coupling strength between the ^{87}Rb atom and the cavity field is $g = 2\pi \times 10.4\text{ MHz}$, which is larger than the cavity field decay rate $\kappa = 2\pi \times 1.4\text{ MHz}$ and the atom dipole decay rate $\gamma = 2\pi \times 1.4\text{ MHz}$ [20, 21, 22]. Thus, in such a cavity the single-mode photon can be considered and the quantum dissipation effect can be also negligible. Finally, in the formalism of second quantization based on the self-consistent Gross-Pitaevskii equations for the two-component BECs, the total Hamiltonian for Fig.1 without rotating-wave approximation can be written as

$$H = H_{ph} + H_{at} + H_{at-at} + H_{at-ph} \quad (1)$$

with

$$H_{ph} = \omega a^\dagger a, \quad (2)$$

$$H_{at} = \sum_{l=1,2} \int d^3\mathbf{r} \{ \Psi_l^\dagger(\mathbf{r}) [-\frac{\hbar^2}{2m} \Delta + V_l(\mathbf{r})] \Psi_l(\mathbf{r}) + \frac{q_l}{2} \Psi_l^\dagger(\mathbf{r}) \Psi_l^\dagger(\mathbf{r}) \Psi_l(\mathbf{r}) \Psi_l(\mathbf{r}) \}, \quad (3)$$

$$H_{at-at} = \int d^3\mathbf{r} q_{1,2} \Psi_1^\dagger(\mathbf{r}) \Psi_2^\dagger(\mathbf{r}) \Psi_1(\mathbf{r}) \Psi_2(\mathbf{r}), \quad (4)$$

$$H_{at-ph} = i\hbar\Omega \int d^3\mathbf{r} [\Psi_1^\dagger(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r}) \Psi_2(\mathbf{r}) + \Psi_2^\dagger(\mathbf{r}) \exp(-i\mathbf{k} \cdot \mathbf{r}) \Psi_1(\mathbf{r})] (a - a^\dagger), \quad (5)$$

where a and a^\dagger are the annihilation and creation operators of the cavity mode with the frequency ω ; $\Psi_l(\mathbf{r})$ is the boson field operator; $V_l(\mathbf{r})$ is the single magnetic trapped potential with the frequencies ω_i ($i = x, y, z$); $q_l = 4\pi\hbar^2\rho_l/m$ is the intraspecies interactions with ρ_l being the intraspecies s -wave scattering length and m being the atomic mass; $q_{1,2} = 4\pi\hbar^2\rho_{1,2}/m$ is the interspecies interactions with $\rho_{1,2}$ being the interspecies s -wave scattering length; Ω is the atom-cavity coupling constant; and \mathbf{k} is the wave vector of the quantized field for the cavity mode.

It has been shown that the well-known two-mode approximation, which is defined as $\Psi_1(\mathbf{r}) = c_1\phi_1(\mathbf{r})$ and $\Psi_2(\mathbf{r}) = c_2\phi_2(\mathbf{r})$ with c_1 and c_2 being the annihilation

boson operators, can be used to simplify the Hamiltonian (1) as

$$H = \sum_{l=1,2} (\omega_l c_l^\dagger c_l + \frac{\eta_l}{2} c_l^\dagger c_l^\dagger c_l c_l) + \chi c_1^\dagger c_1 c_2^\dagger c_2 + \lambda (c_1^\dagger c_2 + c_2^\dagger c_1) (a + a^\dagger) + \omega a^\dagger a, \quad (6)$$

where $\omega_l = \int d^3\mathbf{r} \{ \phi_l^*(\mathbf{r}) [-\frac{\hbar^2}{2m} \Delta + V_l(\mathbf{r})] \phi_l(\mathbf{r}) \}$, $\eta_l = q_l \int d^3\mathbf{r} |\phi_l(\mathbf{r})|^4$, $\chi = q_{1,2} \int d^3\mathbf{r} |\phi_1(\mathbf{r})|^2 |\phi_2(\mathbf{r})|^2$ and $\lambda = i\hbar\Omega \int d^3\mathbf{r} \phi_1^*(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r}) \phi_2(\mathbf{r}) = i\hbar\Omega \int d^3\mathbf{r} \phi_2^*(\mathbf{r}) \exp(-i\mathbf{k} \cdot \mathbf{r}) \phi_1(\mathbf{r})$ are the atom-cavity interacting constant and assumed to be real for the sake of simplicity. However, it should be noticed that this two-mode approximation can only be applied for a small number of condensed atoms[39, 40]. A simple estimate shows that the number of atoms N should satisfy the condition such that $N\rho \leq r_0$, where ρ is a typical scattering length and r_0 is a measure of the trap size. If we take into account large trap size $r_0 = 10\text{ }\mu\text{m}$ and the typical scattering length $\rho = 5\text{ nm}$ [41], the two-mode approximation is valid for $N \leq 2000$.

The Hamiltonian (2) can be further simplified by applying the Schwinger relations defined as $S_x = (c_1^\dagger c_2 + c_2^\dagger c_1)/2$, $S_y = (c_1^\dagger c_2 - c_2^\dagger c_1)/2i$, and $S_z = (c_1^\dagger c_1 - c_2^\dagger c_2)/2$, where the Casimir invariant is $S^2 = N(N/2 + 1)/2$. The eigenvalues m of operator S_z represent the difference $2(N_1 - N_2)$ in the number of atoms in different hyperfine levels, while S_x and S_y take on the meaning of relative phase between two condensates. Besides a trivial constant, the Hamiltonian (6) can be reduced to a generalized Dicke Hamiltonian with a quadratic term

$$H = qS_z^2 + \omega_0 S_z + 2\lambda S_x (a^\dagger + a) + \omega a^\dagger a, \quad (7)$$

where $q = [(\eta_1 + \eta_2)/2 - \chi]$ and $\omega_0 = \omega_1 - \omega_2 + (N - 1)(\eta_2 - \eta_1)/2 \simeq N(\eta_2 - \eta_1)/2$ for the single trapped potential. The Hamiltonian (7) describes the collective quantum dynamics of two-component BECs with few ^{87}Rb atoms inside an ultrahigh finesse optical cavity. It can be seen clearly that the nonlinear quadratic term arises from the mean-field interatomic interactions, which can be controlled by the s -wave scattering lengths of atoms via Feshbach resonance technique. For simplicity, here we assume that the weak nonlinear parameter q depends only on the interspecies interaction, namely, on the parameter χ (or $\rho_{1,2}$) and consider the case of $\omega_0 \geq 0$, that is, $\rho_2 \geq \rho_1$. If the s -wave scattering lengths of atoms are chosen as $\rho_{1,2} = (\rho_1 + \rho_2)/2$, the Hamiltonian (7) can be mapped into the standard Dicke-like Hamiltonian $H_D = \omega a^\dagger a + \omega_0 S_z + 2\lambda S_x (a^\dagger + a)$ [42].

In order to effectively describe the collective dynamical behavior of the Hamiltonian (7), we should evaluate its ground-state energy. Since here the trapped atom number is considered to be of order 10^3 , we can use the mean-field approximation to arrive at the target. Following the procedure of Ref.[43], we firstly employ the well-known Holstein-Primakoff transformation, which is

defined as $S_+ = b^+ \sqrt{N - b^+ b}$, $S_- = \sqrt{N - b^+ b} b$, and $S_z = (b^+ b - N/2)$ with $[b, b^+] = 1$ [36], to rewrite the Hamiltonian (7), apart from a trivial constant, as

$$H = \omega a^+ a + \tilde{\omega}_0 (b^+ b - N/2) + \frac{g}{\sqrt{N}} (b^+ \sqrt{N - b^+ b} + \quad (8)$$

$$\sqrt{N - b^+ b} b)(a^+ + a) + \frac{\nu}{N} (b^+ \sqrt{N - b^+ b})(\sqrt{N - b^+ b} b)$$

where $\nu = -Nq$, $g = \sqrt{N}\lambda$ and $\tilde{\omega}_0 = \omega_0 + q \simeq \omega_0$ since the parameter q is a negligibly small number compared with the parameter ω_0 . Secondly, we introduce the new boson operators by setting $c^+ = a^+ + \sqrt{N}\alpha$ and $d^+ = b^+ - \sqrt{N}\beta$ to describe the collective behavior of the Hamiltonian (8). At last, in terms of boson expansion method based on the introduced operators c^+ and d^+ , the Hamiltonian (8) can be expanded by[37]

$$H = NH_0 + N^{1/2}H_1 + N^0H_2 + \dots \quad (9)$$

with $H_0 = \omega\alpha^2 + \omega_0(\beta^2 - 1/2) - 4g\alpha\beta\sqrt{k} + \nu\beta^2\sqrt{k}$, $H_1 = (-\omega\alpha + 2g\beta\sqrt{k})(c^+ + c) + [\omega_0\beta - (2g\alpha - \nu\beta\sqrt{k})(\sqrt{k} - \beta^2/\sqrt{k})](d^+ + d)$, $H_2 = \omega c^+ c + (\omega_0 + 4g\alpha\beta/\sqrt{k} + \nu k - 3\nu\beta^2)d^+ d + (g\alpha\beta/\sqrt{k} - \nu\beta^2)[(d^+)^2 + d^2] + (g\alpha\beta^3/2k^{3/2})(d^+ + d)^2 + (g\sqrt{k} - \beta^2/\sqrt{k})(c^+ + c)(d^+ + d)$, where $k = 1 - \beta^2$.

The critical transition point can be derived from the condition $H_1 \equiv 0$, which means that for any quantum system the free energy should be fixed at the minimum value. Therefore, two different cases for $\delta > 1$ and $\delta < 1$ with $\delta = \omega\omega_0/N(4\lambda^2 + \omega q)$ must be considered. The corresponding auxiliary parameters α and β are given by

$$\alpha = \begin{cases} 0, & \delta > 1 \\ \frac{\sqrt{N}\lambda\sqrt{1-\delta^2}}{\omega}, & \delta < 1 \end{cases}, \beta = \begin{cases} 0, & \delta > 1 \\ \sqrt{\frac{(1-\delta)}{2}}, & \delta < 1 \end{cases} \quad (10)$$

and the interesting critical transition point is given by $\delta = 1$, namely,

$$q_c = \frac{\omega_0}{N} - \frac{4\lambda^2}{\omega}. \quad (11)$$

Since the many-body interactions in the two-mode approximation produce only small modification of the ground-state properties, the wavefunctions of the macroscopic condensate states for the single magnetic trap can be given, if the acting role of the gravity is omitted, by $\phi_l(\mathbf{r}) = \pi^{-3/4}(d_x d_y d_z)^{-1/2} \exp[-(x^2/d_x^2 + y^2/d_y^2 + z^2/d_z^2)/2]$ with $d_x = \sqrt{\hbar/m\omega_x}$, $d_y = \sqrt{\hbar/m\omega_y}$ and $d_z = \sqrt{\hbar/m\omega_z}$. When the relevant parameters are chosen as $\omega_x = \omega_y = 2\pi \times 290$ Hz, $\omega_z = 2\pi \times 450$ Hz, $m = 1.45 \times 10^{-25}$ kg, $\rho_1 = 3.70$ nm, $\rho_2 = 5.70$ nm, and $N = 1000$, respectively, the parameter ω_0 in the unit of \hbar can be approximately calculated as $\omega_0 \simeq N\hbar^2(\rho_2 -$

$\rho_1)/\sqrt{2\pi}d_x d_y d_z m \simeq 2866$ Hz. If the frequency of the photon and the atom-field coupling strength are considered by $\omega = 4 \times 10^8$ MHz and $\lambda = 2\pi \times 5$ MHz[20, 21, 22], respectively, the critical transition point is evaluated as $q_c = -7$ Hz, whose corresponding interspecies s -wave scattering length is given by $(\rho_{1,2})_c = 7.14$ nm. It seems that this critical parameter $(\rho_{1,2})_c$ can be achieved by current Feshbach resonance technique, which awaits experimental validation.

In the case of $\rho_{1,2} > (\rho_{1,2})_c$ ($\delta > 1$), the ground-state energy is given by $(E_0)_{nor} = -N\omega_0/2$, which depends only on the parameter of the BECs. However, in the case of $\rho_{1,2} < (\rho_{1,2})_c$ ($\delta < 1$), the ground-state energy becomes $(E_0)_{sup} = -N[N(\lambda^2/\omega + q/4)(1 - \delta^2) + \omega_0\delta/2]$, which is dependent of the parameters of both the BECs and the cavity field. The physics may be understood as follows. For the case $\rho_{1,2} > (\rho_{1,2})_c$, the quadric term (qS_z^2) in the Hamiltonian (7) plays an important role, which means that the transition between two condensates is suppressed and therefore no collective excitation happens. However, for the other case $\rho_{1,2} < (\rho_{1,2})_c$, the energy of the term $S_x(a^+ + a)$ in the Hamiltonian (7) dominates, which implies that both the internal Josephson tunneling and the macroscopic collective excitation occur. Therefore, we can call the normal phase when $\rho_{1,2} > (\rho_{1,2})_c$ and the superradiant phase when $\rho_{1,2} < (\rho_{1,2})_c$. Fig.2 shows the scaled ground-state energy E_0/N and its second-order derivative with respect to the intraspecies s -wave scattering length $\rho_{1,2}$ as a function of $\rho_{1,2}$. It can be seen that the second-order derivative of E_0/N possesses a discontinuity at the transition point $(\rho_{1,2})_c$, which clearly illustrates the nature of second-order phase transition (the first-order derivative of E_0/N is continuous at the transition point). Fig.3 shows the scaled ground-state atom population between two condensates

$$\frac{\Delta N}{N} = 2 \langle S_z \rangle = \begin{cases} -\frac{\omega\omega_0}{N(4\lambda^2 + \omega q)}, & \rho_{1,2} < (\rho_{1,2})_c \\ -1, & \rho_{1,2} > (\rho_{1,2})_c \end{cases} \quad (12)$$

and its first-order derivative with respect to the intraspecies s -wave scattering length $\rho_{1,2}$ as a function of $\rho_{1,2}$, which also demonstrates the superradiant phase transition.

In the rest part of this paper we discuss how to observe this predicted phase transition in current experimental setups. In general, the many-body quantum pseudospin state in this quantum system is not accessible to observe quantum phase transition. However, here we propose to detect the direct and striking signatures of the photon field such as the well-measured intracavity intensity $I \propto |\langle a \rangle|^2$ in terms of a heterodyne detector out of the cavity[38]. In terms of the auxiliary parameters α given by Eq.(10), the scaled ground-state intracavity intensity is evaluated as

$$\frac{I}{N} \propto \begin{cases} \frac{\lambda^2 [N^2(4\lambda^2 + \omega q)^2 - \omega^2 \omega_0^2]}{N\omega^2(4\lambda^2 + \omega q)^2}, & \rho_{1,2} < (\rho_{1,2})_c \\ 0, & \rho_{1,2} > (\rho_{1,2})_c \end{cases} \quad (13)$$

Fig.4 shows the scaled ground-state intracavity intensity I/N and its first-order derivative with respect to the intraspecies s -wave scattering length $\rho_{1,2}$ as a function of $\rho_{1,2}$. It is interesting that this predicted quantum phase transition characterized by the non-analyticity of the scaled ground-state intracavity intensity I/N is remarkably of the first-order. When the intraspecies s -wave scattering length $\rho_{1,2}$ approaches the critical value $(\rho_{1,2})_c$, the scaled ground-state intracavity intensity I/N vanishes as

$$\frac{I}{N}[\rho_{1,2} \rightarrow (\rho_{1,2})_c] \sim |\rho_{1,2} - (\rho_{1,2})_c|. \quad (14)$$

Since the diverging characteristic length scale is $\zeta \sim |\rho_{1,2} - (\rho_{1,2})_c|^{-v}$ with $v = 1/2$, the critical exponent for the scaled ground-state intracavity intensity can be derived from $I/N \sim |\rho_{1,2} - (\rho_{1,2})_c|^{zv}$ by $z = 2$,

which shows the universality principle of quantum phase transition[44].

In conclusion, we have analytically discussed the ground-state properties of two-component BECs of ^{87}Rb atoms inside an ultrahigh finesse optical cavity supporting a single-mode photon. It has been shown that the weak interspecies mean-field interaction can give rise to a novel zero-temperature quantum phase transition from the normal to the superradiant phases, which means that the weak microscopic nonlinear interaction can lead to a strong behavior in macroscopic scale. Moreover, we have proposed to observe this predicted quantum phase transition by measuring the direct and striking signatures of the photon field such as the well-measured intracavity intensity $I \propto |\langle a \rangle|^2$ in terms of a heterodyne detector out of the cavity. It is also interesting that this scaled ground-state intracavity intensity I/N is remarkably of the first-order at the transition point $(\rho_{1,2})_c$.

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- [1] THOMPSON R. J., REMPE G. and KIMBLE H. J., *Phys. Rev. Lett.*, **68** (1992) 1132.
 - [2] MCKEEVER J., BOCA A., BOOZER A. D., BUCK J. R. and KIMBLE H. J., *Nature (London)*, **425** (2003) 268.
 - [3] BOOZER A. D., BOCA A., BUCK J. R., MCKEEVER J. and KIMBLE H. J., *Phys. Rev. A*, **70** (2004) 023814.
 - [4] MÜNSTERMANN P., FISCHER T., MAUNZ P., PINKSE P. W. H. and REMPE G., *Phys. Rev. Lett.*, **82** (1999) 3791.
 - [5] HOOD C. J., LYNN T. W., DOHERTY A. C., PARKINS A. S. and KIMBLE H. J., *Science*, **287** (2000) 1447.
 - [6] YE J., VERNOOY D. W. and KIMBLE H. J., *Phys. Rev. Lett.*, **83** (1999) 4987.
 - [7] PINKSE P. W. H., FISCHER T., MAUNZ P. and REMPE G., *Nature (London)*, **404** (2000) 365.
 - [8] VANENK S. J., MCKEEVER J., KIMBLE H. J. and YE J., *Phys. Rev. A*, **64** (2001) 013407.
 - [9] MAUNZ P., PUPPE T., SCHUSTER I., SYASSEN N., PINKSE P. W. H. and REMPE G., *Nature (London)*, **428** (2004) 50.
 - [10] KUHN A., HENNRICH M. and REMPE G., *Phys. Rev. Lett.*, **89** (2002) 067901.
 - [11] MCKEEVER J., *et al.*, *Science*, **303** (2004) 1992.
 - [12] CIRAC J. I., ZOLLER P., KIMBLE H. J. and MABUCHI H., *Phys. Rev. Lett.*, **78** (1997) 3221.
 - [13] PIOVELLA N., COLA M. and BONIFACIO R., *Phys. Rev. A*, **67** (2003) 013817.
 - [14] GASENZER T., ROBERTS D. C. and BURNETT K., *Phys. Rev. A*, **65** (2002) 021605.
 - [15] DEB D. and AGARWAL G. S., *Phys. Rev. A*, **65** (2002) 063618.
 - [16] COLA M. M., PARIS M. G. A. and PIOVELLA N., *Phys. Rev. A*, **70** (2007) 043809.
 - [17] MASCHLER C. and RITSCH H., *Phys. Rev. Lett.*, **95** (2005) 260401.
 - [18] NAGORNY B., ELSÄSSER T. and HEMMERICH A., *Phys. Rev. Lett.*, **91** (2003) 153003.
 - [19] SAUER J. A., FORTIER K.M., CHANG M. S., HAMLEY C. D. and CHAPMAN M. S., *Phys. Rev. A*, **69** (2004) 051804.
 - [20] ÖTTL A., RITTER S., KÖHL M. and ESSLINGER T., *Phys. Rev. Lett.*, **95** (2005) 090404.
 - [21] BOURDEL T., DONNER T., RITTER S., ÖTTL A., KÖHL M. and ESSLINGER T., *Phys. Rev. A*, **73** (2006) 043602.
 - [22] ÖTTL A., RITTER S., KÖHL M. and ESSLINGER T., *Rev. Sci. Instrum.*, **77** (2006) 063118.
 - [23] BRENNECKE F., DONNER T., RITTER S., BOURDEL T., KÖHL M. and ESSLINGER T., *quant-ph:0706.3411* (2007).
 - [24] INOUE S., *et al.*, *Nature (London)*, **392** (1998) 151.
 - [25] GOLDSTERN E. and MEYSTRE P., *Phys. Rev. A*, **55** (1997) 2935.
 - [26] PU H. and BIGELOW N. P., *Phys. Rev. Lett.* **80** (1998) 1134.
 - [27] DENG L., *et al.*, *Nature (London)*, **398** (1999) 218.
 - [28] STRECKER K. E., *et al.*, *Nature (London)*, **417** (2002) 150.
 - [29] CARR L. D. and BRAND J., *Phys. Rev. Lett.*, **92** (2004) 040401.
 - [30] LEE C., and BRAND J., *Europhys. Lett.*, **73** (2006) 321.
 - [31] LEE C., *et al.*, *Phys. Rev. A*, **69** (2004) 033611.
 - [32] LEE C., *et al.*, *Phys. Rev. A*, **64** (2001) 053604.
 - [33] SHLIZERMAN E. and ROM-KEDAR V., *Phys. Rev. Lett.*, **96** (2006) 024104.
 - [34] LEE C., *Phys. Rev. Lett.*, **97** (2006) 150402.

- [35] CHEN G., LIANG J. -Q. and CHEN Z. D., *Europhys. Lett.*, **79** (2007) 10001.
- [36] HOLSTEIN T. and PRIMAOKOFF H., *Phys. Rev.*, **58** (1949) 1098.
- [37] KLEIN A. and MARSHALEK E. R., *Rev. Mod. Phys.*, **63** (1991) 375.
- [38] MCKEEVER J., BUCK J. R., BOOZER A. D. and KIMBLE H. J., *Phys. Rev. Lett.*, **93** (2004) 143601.
- [39] MIBURN G. J., CORNEY J., WRIGHT E. M. and WALLS D. F., *Phys. Rev. A*, **55** (1997) 4318.
- [40] TSUKADA N., GOTODA M., NOMURA Y. and ISU T., *Phys. Rev. A*, **59** (1999) 3862.
- [41] DAVIS K. B., *et al.*, *Phys. Rev. Lett.*, **75** (1995) 3969.
- [42] DICKE R. H., *Phys. Rev.*, **93** (1954) 99.
- [43] EMARY C. and BRANDES T., *Phys. Rev. E*, **67** (2003) 066203.
- [44] SACHSDEV S., *Quantum Phase transitions* (Cambridge University Press, Cambridge) 1999.

Figure Captions

Fig.1 (Color online) Schematic diagram for two-component BECs of few ^{87}Rb atoms inside a high-quality cavity quantum electrodynamics. Initially, two condensates in different hyperfine levels $|F = 1, m_f = -1\rangle$ and $|F = 2, m_f = 1\rangle$ are confined in a time-average, orbiting potential magnetic trap. After BECs are sent into the

optical cavity, the quantized field of the cavity mode can be used to adiabatically control the various transition between these two condensates. The heterodyne detector (HD) out of the cavity can be applied to conveniently detect the well-measured intracavity intensity.

Fig.2 (Color online) The scaled ground-state energy E_0/N versus the intraspecies s -wave scattering length $\rho_{1,2}$. The relative parameters are chosen as $\omega_x = \omega_y = 2\pi \times 290$ Hz, $\omega_z = 2\pi \times 450$ Hz, $m = 1.45 \times 10^{-25}$ kg, $\rho_1 = 3.70$ nm, $\rho_2 = 5.70$ nm, $\omega = 4 \times 10^8$ MHz, $\lambda = 2\pi \times 5$ MHz and $N = 1000$. The critical interspecies s -wave scattering length can be evaluated by $(\rho_{1,2})_c = 7.14$ nm. Insert: the second-order derivative of E_0/N versus with respect to $\rho_{1,2}$ versus $\rho_{1,2}$.

Fig.3 (Color online) The scaled ground-state atom population between two condensates $\Delta N/N$ versus the intraspecies s -wave scattering length $\rho_{1,2}$ with the same parameters as those in Fig.2. Insert: the first-order derivative of $\Delta N/N$ with respect to $\rho_{1,2}$ versus $\rho_{1,2}$.

Fig.4 (Color online) The scaled ground-state intracavity intensity I/N versus the intraspecies s -wave scattering length $\rho_{1,2}$ with the same parameter as those in Fig.2. Insert: the first-order derivative of I/N with respect to $\rho_{1,2}$ versus $\rho_{1,2}$.